

# Engineering Student Misconceptions about Rate and Accumulation Processes

## Preliminary Evidence for the Development of the Rate and Accumulation Concept Inventory

Carli D. Flynn\*, Cliff I. Davidson

Department of Civil and Environmental Engineering  
Syracuse University  
Syracuse, USA  
\*cflynn@syr.edu

Sharon Dotger

Department of Science Teaching  
Syracuse University  
Syracuse, USA

**Abstract** — This paper reports preliminary evidence that many engineering students possess robust misconceptions about rates of change in processes. Exploratory testing on this issue led to the development of the Rate and Accumulation Concept Inventory (RACI), which is intended to assess the extent and types of misconceptions about rate processes. Initial results of this instrument indicate the presence of multiple levels of misconceptions among sophomore level engineering students.

**Keywords**— *concept inventory; rate processes; misconception*

### I. INTRODUCTION

Engineering students often have robust misconceptions that can persist throughout their educational career and may hinder their ability to learn new material [1, 2]. Concept inventories are multiple choice instruments that have been used in several engineering disciplines as a way to provide reliable and valid assessment of students' misconceptions. Some of these inventories include assessments of conceptual frameworks related to rate processes. However, they are usually context specific and do not delve into the mathematical conceptual frameworks that may underlie contextual misconceptions.

The work of Hestenes et al. on the Force Concept Inventory (FCI) established many of the protocols used in concept inventories today [3]. The FCI sought to test physics students' "common sense" ideas regarding everyday phenomena. Extensive work was done in developing the FCI concepts and pilot testing the inventory items. Similar iterative methods have been suggested for developing other concept inventories for science and engineering disciplines [4].

This paper describes the development of the Rate and Accumulation Concept Inventory (RACI). We begin with presentation of exploratory work, which demonstrates the need for the inventory. This is followed by a discussion of the development of the RACI and preliminary results. The paper ends with a discussion of plans for ongoing and future work.

### II. EXPLORATORY WORK

#### A. Objectives

The primary objective of the exploratory study was to determine whether there are robust student misconceptions that may impede student learning of applied engineering concepts related to water flow processes, and if misconceptions are present, to identify them. Lecture and class activities would then be tailored as "intervention" methods to overcome prevailing misconceptions. The context of study was an urban hydrology unit that is part of a sophomore-level engineering course. Several types of student understanding were considered, including equation based, graphical, and illustrative or descriptive understanding. It was hypothesized that a lack in any of the three types of understanding of a fundamental engineering conceptual framework would impede student understanding of advanced engineering concepts.

#### B. Design

Two survey instruments were developed in order to assess student understanding of two fundamental engineering conceptual frameworks: first order calculus and mass flow. "Survey Instrument 1" aimed to assess the students' prerequisite conceptual frameworks, as the instrument made use of concepts from only prerequisite coursework. For example, first order calculus concepts were assessed using the case of a decelerating airplane, and mass flow concepts were assessed using natural water flows to a lake (see Figure 1 and Table I for prompts and questions). "Survey Instrument 2" included applied engineering concepts of hydrologic flows that were assumed to be unfamiliar to most of the students at the time of initial testing. These concepts included groundwater flow and water flows on a green roof (see Figure 2 and Table II for prompts and questions).

Both Survey Instrument 1 and Survey Instrument 2 were administered as a pre-intervention to assess the students'

existing conceptual frameworks at the beginning of the course. The intervention in this context refers to lectures and class activities to familiarize the students with hydrologic engineering concepts; there was no attempt to review the fundamental principles of first order calculus and mass flow. Survey Instrument 2 was administered a second time as a post-intervention survey.

Each survey was administered to approximately 90 sophomore engineering students enrolled in the course. They were administered within normal class periods, and a 25-minute time restriction was applied for each survey. Survey Instrument 1 was administered within the first week of the course, and Survey Instrument 2 was administered during the sixth week of the course and at the conclusion of the course. The resulting data were compared to identify misconceptions of fundamental principles that continued to exist in the applied engineering concepts after instruction took place.

Additional research methods included video and audio recordings of three groups of five students completing activities that were designed to assess the learning of two topics, namely groundwater flow and water flows on a green roof (see Figures 3 and 4, respectively). The recordings captured nuances in communication styles and thinking patterns associated with the written work. Methods of data analysis followed a grounded theory approach. Grounded theory was chosen since little research has been conducted regarding engineering students' collaborative understanding of flow processes. Analysis methods are based on several well-supported texts on this approach [5, 6]. Transcriptions were made for the collected video and audio data for the three groups. ATLAS.ti was used to compile and analyze the transcription data. First, open line-by-line codes were assigned to each statement the students made. After reviewing the codes and the transcriptions several times, two memos were produced, summarizing the initial findings from the two activities. Focused codes were then assigned to similar types of open codes. A final memo was then developed describing the overall findings on student ideas about water flow and the effectiveness of the learning activities.

**Calculus** - An airplane has just touched down with  $t = 0$  corresponding to the instant the wheels first touch the runway as the plane lands. This is also the location where  $x = 0$ . The velocity of the plane at  $t = 0$  is  $V_0$ . The plane is subjected to a constant deceleration  $-a_0$ . Given:

$$V = \frac{dx}{dt}, \quad a = \frac{dV}{dt} = \frac{d^2x}{dt^2}$$

**Water Flow** - Consider the following variables for the water balance for a lake:  $S$  (depth of lake),  $P$  (precipitation rate to lake),  $E$  (evaporation rate from lake), and  $RO$  (runoff rate measured from streams to lake). Assume no other flows into or out of the lake.

Fig. 1. Survey Instrument 1 Question Prompts

**Groundwater Flow** - Darcy's law provides an accurate description of the rate of ground water flow ( $Q$ ) determined by head loss ( $dh$ ), the hydraulic conductivity ( $K$ ), the cross-sectional area ( $A$ ), and the horizontal distance of the flow ( $dL$ ). Given:

$$Q = -KA \left( \frac{dh}{dL} \right)$$

**Green Roof Water Flow** - Consider the following variables for the water balance for a green roof:  $S$  (depth of water stored on green roof),  $P$  (precipitation rate),  $ET$  (evapotranspiration rate), and  $Q_{\text{drain}}$  (rate of rainwater flow into drain).

Fig. 2. Survey Instrument 2 Question Prompts

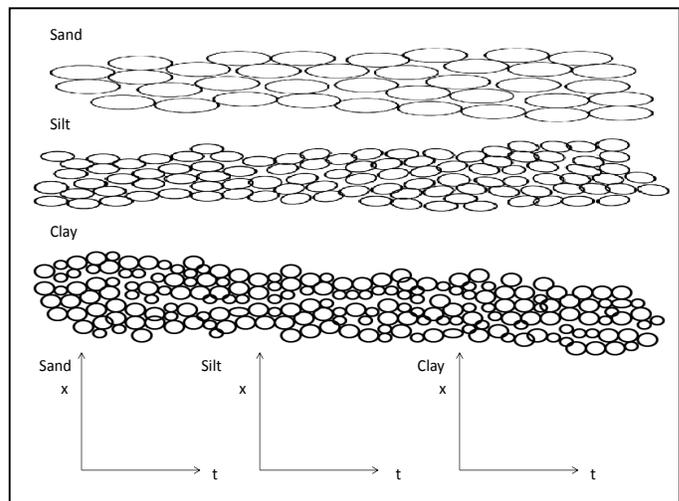


Fig. 3. Groundwater Flow Activity

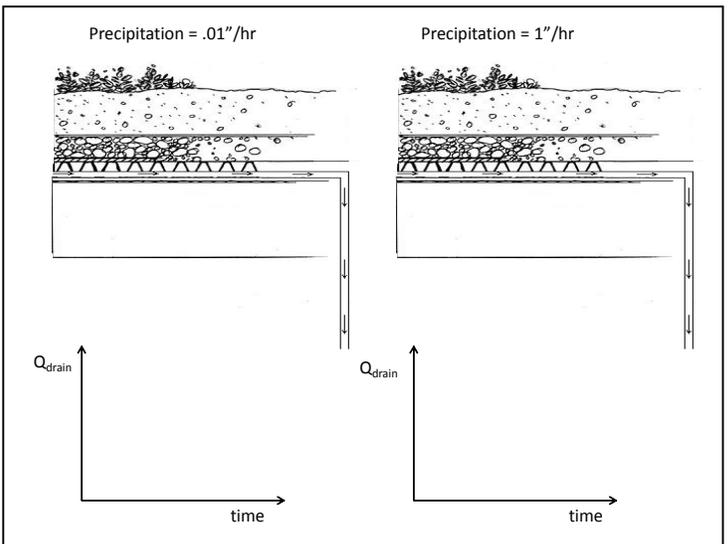


Fig. 4. Green Roof Water Flow Activity

TABLE I. SURVEY INSTRUMENT 1 QUESTIONS

	Identifier	Question
First order calculus	C1	Write an expression for $x(t)$ only in terms of $a_0$ , $V_0$ and $t$ .
	C2, C3	Graph the curve of $x(t)$ and $V(t)$
	C4	How would your curve for $V(t)$ change if the deceleration rate were increasing instead of constant?
Basic Water Flow	W1	Using the given terms, write an equation for $dS/dt$ .
	W2	Draw a picture of the lake using arrows to represent the water flows for each term in the equation for $dS/dt$ . Show whether the flow is vertical up, vertical down, or not vertical.
	W3	The area around the lake above experiences urban development. The runoff rate $RO$ for the watershed increases exponentially over time as more soil becomes covered with concrete and asphalt due to urban growth. Assume $P$ is constant, and $E$ is constant over time. Plot a curve of $S$ versus time.
	W4	Describe in one sentence how your plot would change if the precipitation rate also increased over time.

TABLE II. SURVEY INSTRUMENT 2 QUESTIONS

	Identifier	Question
Groundwater Flow	GW1	Consider a confined aquifer with a source of recharge, where $K$ is 50 m/day, head loss is 5m, the horizontal distance of flow is 100m, and the cross-sectional area is 2,000 $m^2$ . What is the total volume of flow (in $m^3$ ) to the aquifer in 5 days?
	GW2	In each of the boxes, use ten circles to depict ten particles of each soil type, differentiating the size and void space for each type. <sup>a</sup>
	GW3	Using the axis provided, plot three curves for the relative time it would take the same quantity of water to travel through an equal distance of each soil type. Label each curve according to the soil type.
	GW4	What other factors (besides soil related factors) will influence the velocity of groundwater flow?
Green Roof Water Flows	GR1	Using the given terms, write an equation for $dS/dt$ .
	GR2	Make a drawing of the water flow represented by each term in the equation for $dS/dt$ . Show whether the flow is vertical up, vertical down, or not vertical.
	GR3	Plot a curve of $Q_{\text{drain}}$ versus time for a small precipitation rate (0.1"/hour) and a larger precipitation rate (1"/hour). The initial condition of the soil media is saturated.
	GR4	What terms in your equation from question 1 would change if the green roof had twice as many plants?
	GR5	How would this affect your plots in question 3?

<sup>a</sup> Three blank boxes were provided, respectively labeled "Sand" "Silt" and "Clay"

TABLE III. MEAN SCORES FOR FIRST ORDER CALCULUS RESPONSES

	C1	C2	C3	C4	Total
Pre-intervention (N=80)	2.03 (68%)	2.34 (78%)	2.56 (85%)	1.75 (58%)	8.68 (72%)

TABLE IV. MEAN SCORES FOR BASIC WATER FLOW RESPONSES

	W1	W2	W3	W4	Total
Pre-intervention (N=80)	2.01 (67%)	1.90 (63%)	1.93 (64%)	1.83 (61%)	7.66 (64%)

TABLE V. MEAN SCORES FOR GROUNDWATER FLOW RESPONSES

	GW1	GW2	GW3	GW4	Total
Pre-intervention (N=58)	2.66 (89%)	1.94 (65%)	2.24 (75%)	1.60 (53%)	8.45 (70%)
Post-intervention (N=85)	2.28 <sup>a</sup> (76%)	1.97 (66%)	2.41 (80%)	1.58 (53%)	8.25 (69%)

<sup>a</sup> Indicates two tailed t-test showed significant change from the pre-intervention test to the post-intervention test at the  $p < 0.01$  level.

TABLE VI. MEAN SCORES FOR GREEN ROOF FLOW RESPONSES

	GR1	GR2	GR3	GR4	GR5	Total
Pre-intervention (N=58)	1.68 (56%)	1.84 (61%)	1.17 (39%)	1.74 (58%)	0.93 (31%)	7.38 (46%)
Post-intervention (N=85)	2.47 <sup>a</sup> (82%)	2.47 <sup>a</sup> (82%)	1.34 (45%)	1.69 (56%)	0.86 (29%)	8.83 <sup>a</sup> (55%)

<sup>a</sup> Indicates two tailed t-test showed significant change from the pre-intervention test to the post-intervention test at the  $p < 0.01$  level.

### C. Results

Rubrics were developed to grade each question response on a scale of 0-3, with a score of "0" representing either no response or an off-topic response, and a "3" representing a completely correct response. A score of "2" included responses that were partially correct with only one incorrect component, while a score of "1" included responses that were completely incorrect or those with more than one incorrect component. The point equivalencies changed with the different questions and the variety of answers. The rubrics went through several iterations of revisions as specific patterns and variations in student responses were identified.

To assure the reliability of the rubric, 20 randomly selected surveys from each of the pre-intervention instruments were graded by a second grader, representing approximately 29% of the pre-intervention assessments. The average degree of agreement for each question was 93% for Survey Instrument 1 and 86% for Survey Instrument 2. All disagreements in individual questions scores were a difference of one point, with the exception of two instances on Survey Instrument 2 for questions GR4 and GR5 that had 2 point differences in scoring. The rubrics for these two questions were revised in order to account for a large range in student responses. Other

minor rubric revisions were made after discussions among the graders and authors on the discrepancies in grading.

### *Survey Instrument 1*

Results are summarized in Tables III and IV. The sum of the student scores for the first order calculus questions resulted in a mean total score of 8.68, or 72%. Large ranges for score on the first order calculus questions C1-C4 suggest that the students were somewhat more comfortable with the graphing questions than with the equation or open-ended questions. The basic water flow questions resulted in a mean score of 7.66, or 64%. The consistency among these scores in Table IV suggests that overall student understanding of this problem was not influenced by the type of response required for the question (i.e. equation, graphing, mental model or open ended).

### *Survey Instrument 2*

Results are summarized in Tables V and VI. The sum of the student scores for the groundwater flow questions resulted in a pre-intervention mean score of 70% and a post-intervention mean score of 69%. Relatively consistent question scores suggest that student conceptions of this problem remained unchanged after course instruction. The only significant change in scores was actually a decrease in scores for GW1, which was an equation based calculation problem.

The sum of the student scores for the green roof water flow questions resulted in a pre-intervention mean score of 7.38, or 46% and a post-intervention mean total score of 8.83, or 55%. This increase in scores is shown to be significant at the  $p < 0.01$  level using an unpaired t-test. As seen in Table VI, the greatest increase in scores is found in the first two questions of this problem, GR1 and GR2, which were the equation and mental model questions. Scores remained lower for GR3, which prompted students to graph the rate of water flow into a roof drain over time, despite instruction including an activity which closely mirrored this survey question. This suggests that students continued to struggle with the graphing of a water flow rate even after course instruction.

### *Groundwater Flow and Green Roof Water Flow Activities*

Results from the analysis of the transcriptions of student work indicated that the groundwater flow activity failed to elicit in-depth discussions on groundwater flow. More often, students debated the path they were supposed to take, viewing the activity as a maze. Discussion often turned to what the students thought were the desired or "correct" results of the activity. One group of students filled in assumed answers without completing the activity instructions. Finally, students spent the majority of time on organizing the steps to complete the activity rather than discussing the results.

The green roof activity elicited a lot of student discussion surrounding the idea of saturation points, including the

possibility of a specific time when water will begin to rush into the drain. Some of this confusion was linked back to the prompt delivered by the instructor, as students tried to recall the language used by the instructor for clues for what the water flow rate curve might be. Student ideas of saturation are strongly linked to "linear relationships" of water flow. The term "linear" is also used to describe maximum rates or constant rates on graphs. For instance, one student states:

*"I mean, if it's a constant .01 inches/hour then once it gets saturated then all that .01 inch/hour is going to go through so I think it would be linear."*

Other captured ideas suggest confusion between rate processes and accumulation processes. For instance, several students state that the rate of water flowing into the roof drain could be represented by an upward curve with no maximum, which suggests that they may have been representing the total amount of water accumulated over time rather than flow rate into the drain. Other captured ideas suggest misconceptions about factors influencing the rate of water flow into the drain (e.g., the need for soil medium to be fully saturated for water to enter the drain). For instance, one student considers the influence of drain capacity on the flow rate into the drain:

*"...we thought it will level out because the drain can only hold so much after a little while."*

Some of this confusion would likely be mitigated through clearer task design, particularly in the prompt delivered by the instructor. It is also interesting that the concepts of "tank flow" and "pipe flow" are mentioned by some students, suggesting that the students are using several learned concepts from the course to investigate the new problem context presented in this activity.

## III. DEVELOPMENT OF THE RATE AND ACCUMULATION CONCEPT INVENTORY

### *A. Objectives*

The primary objective of the RACI is to identify both the mathematical and applied conceptual frameworks that underlie student misconceptions about rate and accumulation processes. Three sections of conceptual understanding were developed in the inventory: (1) first order calculus, (2) mass flow, in particular water flow, and (3) heat transfer.

### *B. Design*

All questions developed for this inventory were posed in either a multiple choice or open-ended format. Most open-ended questions required the student to describe how they arrived at their response to the previous question. At the end of each set of questions related to a single prompt, students were asked to assess the level of confidence they held in answering the questions. Both of these additional data sets allowed for a greater understanding as to how students were interpreting the inventory questions.

Three categories of questions were included in the inventory. The Mathematical and Mass Flow categories

include original inventory items developed in this study, while the heat transfer inventory items were taken directly from the Heat and Energy Concept Inventory (HECI), developed by Prince et. al. with the author's permission [2].

#### 1) *Mathematical* :

Mathematical concepts are considered in the RACI, as solving rate and accumulation problems require students to interpret the meaning of a function that models a dynamic situation. This ability, called *covariational reasoning*, is essential for representing and interpreting the changing nature of quantities in a wide array of dynamic situations [7].

Three sets of questions were developed to assess mathematical concepts. Each question set draws from first order calculus concepts, as these were identified as the most relevant to rate and accumulation processes. The first two questions are based on problems from an introductory calculus textbook [8]. These questions were developed to assess students' ability to interpret a phenomenon and its associated graphical representation. The third question is based on previous work on assessing students engaging in a covariational tasks [7]. Students were asked to produce graphs based on the change in height and volume over time as water is being poured into a spherical shaped bottle with a rectangular neck. The format of this problem was left as an open-ended question since previous efforts in this study had not investigated concepts related to covariational reasoning.

#### 2) *Mass Flow* :

This misconception area stems from the findings of the exploratory work demonstrating student difficulty in distinguishing between factors that affect the rate at which water flows through a system and the total amount of water flow. Four sets of questions were developed to assess conceptual understanding related to water flow processes. One question was developed to assess the concept of hydrostatic equilibrium, using two bowls of water connected by a pipe. Two sets of questions were developed to assess concepts related to the effect of water height on the rate of water flow. The fourth set of questions addresses concepts on the effect of porosity on water flow .

#### 3) *Heat Flow* :

Only the inventory items from the HECI under the subcategory of "rate vs. amount" were considered, as these were the most relevant to the goals of the RACI. One set of two questions in HECI "rate vs. amount" category are actually mass flow questions, designed as an analog to the heat transfer problems.

### IV. ONGOING AND FUTURE WORK

The RACI will be administered twice in a sophomore level engineering class in order to assess the students' conceptual

frameworks at the beginning and end of the course. The instrument will be administered during normal class periods to all students (approximately 80) enrolled in the course.

#### A. *Development of distractors*

The purpose of the open-ended questions in the current version of the RACI is to collect a range of student reasoning for each question. Incorrect responses will be categorized according to the type of misconception suggested in the students' work. These misconceptions will then be developed into multiple-choice responses known as *distractors*, which are designed to capture patterns of incorrect conceptual reasoning a student may use when answering a question. Distractors will replace the open ended question responses in future versions of the RACI.

#### B. *Interviews*

Interviews will be conducted to further assess student responses, in particular the open ended responses. The interviews are designed to be semi-structured, 20 minute interviews held within a week of the students' completion of the inventory. The option to participate as an interview subject will be open to all students in the course.

### V. SUMMARY

This paper discusses efforts that have been made to study and identify engineering student misconceptions that may impede learning of applied engineering concepts related to water flow processes. Results from these efforts suggest the existence of robust misconceptions about rate and accumulation processes among sophomore engineering students. This has prompted the development of the Rate and Accumulation Concept Inventory (RACI) in order to assess conceptual understanding of the fundamental concepts related to these processes. Development of this instrument is an ongoing and iterative process that will likely go through several versions in order to establish reliability and validity.

The conclusions drawn from this study have certain limitations that should be acknowledged. The sample of students that took the surveys is from a single class in a single institution. Thus, many of these findings may be unique to this particular population of students. Future studies will try to use larger random samples of engineering students whenever feasible.

### ACKNOWLEDGMENTS

The authors would like to thank undergraduate researchers Rachael Thomas, LaVerne Sessler and Meredith Sullivan, graduate researchers Peter Mkhathswa, Jessica Whisher-Hehl, and AnnMarie O'Neil, and Dr. Helen Doerr for their help in this project.

### REFERENCES

- [1] R. T. Streveler, T. Litzinger, R. Miller and P. Steif, "Learning conceptual knowledge in the engineering sciences: Overview and future research directions," *Journal of Engineering Education*. vol. 97, pp. 279–294, 2008.

- [2] M. Prince, M. Vigeant, and K. Nottis, "Development of the Heat and Energy Concept Inventory: Preliminary results on the prevalence and persistence of engineering students' misconceptions," *Journal of Engineering Education*. vol. 101, pp. 421–438, July 2012.
- [3] D. Hestenes, M. Wells, and G. Swackhamer. "Force Concept Inventory," *The Physics Teacher*. vol. 30, pp. 141-158, March 1992.
- [4] J. Richardson, "Concept inventories: Tools for uncovering STEM students' misconceptions," *Invention and Impact: Building Excellence in Undergraduate Science, Technology, Engineering and Mathematics (STEM) Education*. Washington, DC: AAAS. pp. 19-25. 2004.
- [5] K. Charmaz. *Constructing Grounded Theory: A Practical Guide through Qualitative Analysis*. Thousand Oaks, CA: Sage Publishers. 2006.
- [6] J.W. Creswell. *Qualitative Inquiry and Research Design: Choosing Among Five Approaches* 3rd ed. Thousand Oaks, CA: Sage Publishers. 2013.
- [7] M. Carlson, S. Jacobs, E. Coe, S. Larsen, and E. Hsu. "Applying covariational reasoning while modeling dynamic events: A framework and a study." *Journal for Research in Mathematics Education*. vol. 22, pp. 352-278. 2002.
- [8] W. Briggs and L. Cochran, *Calculus*, 1st ed. Boston: Pearson, 2010, pp. 324-325.